

The Inflatino Problem in Supergravity Inflationary Models

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Abstract

We consider the potential problems due to the production of inflatinos and gravitinos after inflation. Inflationary models with a single scale set by the microwave background anisotropies have a low enough reheat temperature to avoid problems with the thermal production of gravitinos. Moreover, the nonthermal production of gravitinos has been shown to be sufficiently small if the sector ultimately responsible for supersymmetry breaking is coupled only gravitationally to the inflationary sector. Still, in some models, inflatinos can be created during preheating with a substantial abundance. The main contribution to the gravitino abundance may thus come from their decay into the inflaton, or into its scalar partner, as well as from the inverse processes. We show that this production needs to be strongly suppressed. This suppression can be realized in the simplest scenarios which typically have a sufficiently high inflationary scale.

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1 Introduction

It is well known that in the particle spectrum of the minimal supersymmetric standard model (the MSSM with R-parity conservation), the lightest stable particle (LSP) is an excellent cosmological dark matter candidate [1, 2]. Indeed, in the constrained version of the MSSM based on minimal supergravity, one can easily satisfy all of the recent accelerator constraints, and at the same time have an LSP with an acceptable and significant cosmological relic density [3]. However, it is also well known that in these same models, there is a potential problem with the abundance of gravitinos in the early Universe [4].

Inflationary models are also subject to potential gravitino problems. If the Universe reheats to sufficiently high temperatures, gravitinos will be produced thermally with excessive abundances [2, 5, 6]. For a gravitational decay of the inflaton field the reheat temperature is directly related to the inflationary scale. When this scale is fixed to match the observed size of density fluctuations, the reheat temperature is typically low enough to avoid thermal overproduction. Recently, the non-thermal production of gravitinos has also been considered [7]. It has been shown [8] that in this case too, gravitinos are produced with a sufficiently low abundance, provided the inflationary sector and the one responsible for supersymmetry breaking are distinct and only gravitationally coupled.

The nonthermal production is much more relevant for the fermionic partner of the inflaton, the inflatino, which in some cases can be produced with a significant abundance. Thus, gravitinos may be overproduced through inflatino decay, if the channel $\text{inflatino} \rightarrow \text{inflaton}$ (or its scalar partner) + gravitino is kinematically allowed. Depending on the relative masses of the inflaton and inflatino, significant production may be expected instead by the inverse process. This is particularly true if the inflaton sector is only gravitationally coupled to matter, since in this case the above decays will have a rate comparable to the one generating the thermal bath. Indeed, in such a scenario, the decay channels into gravitinos need to be strongly suppressed. We discuss the possible kinematic suppression of this channel and derive a strong upper bound on the scale of inflation. We further show that this bound is satisfied by the simple single scale (supergravity) inflationary models.

We begin by reviewing the gravitino problem with respect to both thermal and non-thermal production. We focus on simple models of inflation with a single scale set by the microwave background anisotropies. In section 3, we formulate the inflatino problem and discuss solutions for both the gravitational and non-gravitational decay of the inflaton. This analysis is generalized in section 4, where we discuss the merits of lowering the scale of inflation. In section 5, we consider two specific models of inflation: 1) a model of new inflation

based on supergravity, and 2) chaotic inflationary models. A discussion and concluding remarks are made in section 6.

2 The Gravitino Problem

Unless very massive, $m_{3/2} \gtrsim 20$ TeV, gravitinos would disrupt the successful predictions of big bang nucleosynthesis. The argument is relatively simple: because of their late gravitational decays, gravitinos dominate the energy density of the Universe which becomes matter dominated with a Hubble expansion rate given by $H \sim m_{3/2}^{1/2} T^{3/2} / M_P$, where M_P is the (reduced) Planck mass. Gravitino decay occurs when the decay rate $\Gamma_{3/2} \sim m_{3/2}^3 / M_P^2 \sim H$ or at $T_D \sim m_{3/2}^{5/3} / M_P^{2/3}$. Subsequently, the Universe is reheated to a temperature $T_R \sim \rho(T_D)^{1/4} \sim m_{3/2}^{3/2} / M_P^{1/2}$. The limit on the gravitino mass is obtained by requiring $T_R \gtrsim 1$ MeV, thus allowing big bang nucleosynthesis to occur in a radiation dominated Universe.

However, since the gravitino abundance is diluted during inflation [9], this simple constraint is significantly altered. In fact, the constraint should be expressed as a function of the gravitino mass and abundance after *inflationary* reheating. The most restrictive bound on their number density comes from the photo-destruction of the light elements produced during nucleosynthesis [10]

$$n_{3/2}/s \lesssim 10^{-13} (100 \text{ GeV} / m_{3/2}) \quad (1)$$

for lifetimes $> 10^4$ sec. The thermal production of gravitinos regenerated after inflation has been estimated [2, 5, 9, 10]

$$n_{3/2}/s \sim (\Gamma/H)(T_{3/2}/T_\gamma)^3 \sim \alpha N(T_R)(T_R/M_P)(T_{3/2}/T_\gamma)^3 \quad (2)$$

where $\Gamma \sim \alpha N(T_R)(T_R^3/M_P^2)$ is the production rate of gravitinos, N is the number of degrees of freedom, and the ratio $(T_{3/2}/T_\gamma)^3$ accounts for the dilution of gravitinos by the annihilations of particles between T_R and nucleosynthesis. From the gravitino regeneration rate one can derive a bound on T_R

$$T_R \lesssim 10^9 \text{ GeV} (100 \text{ GeV} / m_{3/2}) \quad (3)$$

For recent discussions on this bound, see [11].

The significance of the above constraint depends on the model of inflation. We assume for simplicity that inflation is governed by a single scale such that the inflaton potential is of the form $V(\phi) = \Delta^4 P(\phi/M_P)$, where $P(\phi/M_P)$ is some suitable function which generates inflation. The scale Δ can be related to the size of large scale fluctuations measured by

COBE [12]. In general this relation is dependent in the choice of P , so that different values for Δ can be considered. However, several of the proposed models typically give [13]

$$\frac{\delta\rho}{\rho} \simeq \frac{H^2}{10\pi^{3/2}\dot{\phi}} \simeq O(100)\frac{\Delta^2}{M_P^2} \quad (4)$$

which, once related to the COBE results, implies [14]

$$\frac{\Delta^2}{M_P^2} = \text{few} \times 10^{-8} \quad (5)$$

Fixing (Δ^2/M_P^2) has several general consequences for inflation [15]. For example, the Hubble parameter during inflation becomes $H^2 \simeq (\Delta^4/3M_P^2)$, leading to $H \sim 10^{-7}M_P$. The duration of inflation is $\tau \simeq M_P^3/\Delta^4$, and the number of e-foldings of expansion is $H\tau \sim (M_P^2/\Delta^2) \sim 10^9$. If the inflaton decay rate goes as $\Gamma \sim m_\phi^3/M_P^2 \sim \Delta^6/M_P^5$, the Universe recovers at a temperature $T_R \sim (\Gamma M_P)^{1/2} \sim \Delta^3/M_P^2 \sim 10^{-11}M_P \sim 10^8$ GeV. This low reheating temperature appears to be safe with regards to the gravitino limit (3) discussed above.

Recently, considerable attention has been focused on the possible non-thermal production of gravitinos after inflation [7, 8]. Fermionic quanta can be created [16] in significant amounts during the first stages of reheating (preheating). When applied to supergravity models, preheating will lead to the non-perturbative production of the fermionic partner of the inflaton, the inflatino, and in general, of any other fermion which is strongly coupled to the inflaton field. If, for example, there is substantial mixing between the inflatino and the longitudinal component of the gravitino, the goldstino, preheating may result in an overproduction of gravitinos. As is well known, the goldstino is a linear combination of the fermionic partners of the scalars responsible for supersymmetry breaking. During inflation and the beginning of reheating, supersymmetry is mainly broken by the inflaton implying a strong correspondence between the inflatino and goldstino at this early stage. However, this correspondence does not necessarily hold at late times, since supersymmetry may be broken by other fields in the true vacuum of the theory. In this case, the final gravitino abundance will be much smaller than the inflatino abundance [8]. Indeed, this is the case in most models as it is natural to distinguish between inflation and supersymmetry breaking due to the very different energy scales associated with the two phenomena. The relic abundance of gravitinos will thus ultimately be related to the strength of the coupling between these two sectors in a given model.

If the scale of inflation, Δ , is much larger than the other scales in the theory, the nonthermal production of inflatinos can be accurately computed by just considering the inflationary

sector. Typically, particle creation occurs during the very first few oscillations of the inflaton on a timescale which is the inverse of the inflaton mass, $m_\phi \sim \Delta^2/M_P$. Most of the inflatinos are produced with momenta $k \lesssim m_\phi$, and hence their initial number density is approximately given by [7]

$$n_{\tilde{\phi}} \sim 0.01 m_\phi^3 \sim 0.01 \left(\frac{\Delta^2}{M_P} \right)^3 \quad (6)$$

The final inflatino abundance is $Y_{\tilde{\phi}} \equiv n_{\tilde{\phi}}/s$ where $s \sim \rho^{3/4}$ is the entropy density at the time of inflaton decay. For now, let us generalize our previous consideration of inflaton decay, and assume only that the decay is given by Γ (rather than assuming specifically a gravitational decay). In this case, if inflaton oscillations begin when the scale factor is $R = R_\phi$, the energy density in oscillations is $\rho_\phi \simeq m_\phi^2 M_P^2 (R_\phi/R)^3$ and $H \simeq m_\phi (R_\phi/R)^{3/2}$. Decays occur when $H \simeq \Gamma$, or when $R = R_{d\phi} = (m_\phi/\Gamma)^{2/3} R_\phi$. During inflaton oscillations, the inflatino number density also scales as R^{-3} so that at the time of decay, $n_{\tilde{\phi}} \sim 0.01 m_\phi^3 (R_\phi/R_{d\phi})^3 \sim 0.01 m_\phi \Gamma^2 \sim 0.01 \Delta^2 \Gamma^2 / M_P$. The entropy density is also easily computed at decay, $s \sim (\Gamma^2 M_P^2)^{3/4} \sim T_R^3$. Thus for a massive inflaton, one finds

$$Y_{\tilde{\phi}} \sim 0.01 \left(\frac{\Delta^2 \Gamma^{1/2}}{M_P^{5/2}} \right) \sim 0.01 \left(\frac{\Delta}{M_P} \right)^3 \left(\frac{T_R}{\Delta} \right) \quad (7)$$

We see, therefore, that a late inflaton decay (low T_R) results in a small inflatino abundance. This is due to the fact that the energy density stored in the coherent oscillations of a massive inflaton redshifts as matter ($\rho \sim R^{-3}$), so that the quantity $n_{\tilde{\phi}}/\rho^{3/4}$ decreases with time.

For the case of a gravitational decay, if we take Δ from eq. (5) and its corresponding reheat temperature $T_R \sim 10^8$ GeV, we find a negligible inflatino abundance $Y_{\tilde{\phi}} \lesssim 10^{-19}$. For a massless inflaton the final abundance will be significantly larger, due to the stronger decrease of the inflaton energy density. For example, for a $V \sim \phi^4$ potential, the energy density of ϕ redshifts as radiation, and the last factor T_R/Δ is absent in the final abundance (7). In this case, the final inflatino abundance may be as large as $Y_{\tilde{\phi}} \simeq 10^{-12}$.

The calculation of the nonthermal production of the longitudinal gravitino component in models with more than just the inflationary sector [8] is more complicated. The mixed inflatino–gravitino system is very involved, and analytical solutions for the gravitino abundance are still lacking. A consistent quantization of the system, with an accurate definition of the occupation numbers of the fermionic eigenstates, is however available. For the case in which the sectors responsible for inflation and supersymmetry breaking are coupled only gravitationally, numerical computations show that gravitino production is restricted to a safe level. The longitudinal gravitinos are created on a physical timescale of the order $m_{3/2}^{-1}$, with a physical typical momentum comparable with $m_{3/2} \ll \Delta$.

3 The Inflatino problem

In the previous section, we have shown that if we restrict the inflationary model so that it is coupled only gravitationally to the other sectors of the theory, both the thermal and the nonthermal production of gravitinos are reduced to a safe level. We have also seen that the nonthermal production of inflatinos is model dependent and can be substantial. If this is the case, and if the inflatino is heavier than the inflaton, the subsequent gravitational decays of the inflatino (to an inflaton and gravitino) could lead to an overproduction of gravitinos (other decays of the inflatino in some specific models are discussed in [17]).

On the contrary, when the inflaton is heavier than the inflatino, then the decay channel $\phi \rightarrow \tilde{\phi} + \tilde{G}$ may be problematic if kinematically allowed [5]. The density of gravitinos produced by inflaton decays is easily estimated. By comparing the number density of inflatons just prior to decay, $n_\phi \sim m_\phi M_P^2 (R_\phi/R_{d\phi})^3 \sim m_\phi^5/M_P^2$ to the entropy density just after decay $s \sim m_\phi^{9/2}/M_P^{3/2}$, one finds $n_\phi/s \sim (m_\phi/M_P)^{1/2} \sim \Delta/M_P$. (It is also convenient to write $n_\phi/s \sim (\Gamma M_P)^{1/2}/m_\phi \sim T_R/m_\phi$). If the branching fraction for inflaton decays to $\tilde{\phi} + \tilde{G}$ is $1/N$, then the resulting gravitino abundance is

$$Y_{3/2} = \frac{n_{3/2}}{s} \sim \frac{\Delta}{NM_P} \sim 10^{-6} \quad (8)$$

in clear violation of the bound (1). This is what we call the inflatino problem. We note that, for a purely gravitational inflaton decay, the limit from direct production is stronger than that due to thermal production. Indeed, by requiring $Y_{3/2} \lesssim 10^{-13}$, eq. (8) yields the constraint $\Delta \lesssim 10^{-11} M_P$ on the inflationary scale, which is stronger than the bound from the thermal production of gravitinos. Requiring $T_R \lesssim 10^9$ GeV, and noting that for gravitational decays, $T_R \sim (\Gamma M_P)^{1/2} \sim \Delta^3/M_P^2$, one finds $\Delta \lesssim 10^{15}$ GeV.

One may hope to relax this difficulty by allowing for non-gravitational couplings of the inflaton to matter. For example, if we suppose that the decay rate to matter is given by $\Gamma_m \sim g^2 m_\phi$, then the gravitino abundance will be suppressed, $Y_{3/2} \sim (\Gamma_G/\Gamma_m) n_\phi/s$ where $\Gamma_G \sim m_\phi^3/M_P^2$ is the gravitational decay rate. In this case, decays occur earlier and $n_\phi/s \sim g(M_P/m_\phi)^{1/2} \sim gM_P/\Delta$. The gravitino abundance produced by inflaton decays is now

$$Y_{3/2} \sim \frac{\Delta^3}{gNM_P^3} \quad (9)$$

Therefore, the constraint on Δ from the gravitino abundance becomes $\Delta \lesssim \text{few} \times 10^{-4} g^{1/3} M_P$. However, for stronger than gravitational decays ($g > \Delta^2/M_P^2$), the constraint from the thermal production of gravitinos becomes more significant. From $T_R \sim g\Delta$, one finds the limit

$\Delta \lesssim 10^9 \text{ GeV}/g$. The combination of the two bounds from the thermal and the direct gravitino production is weakest when $g \sim 10^{-15/4}$. For this value, the limit $\Delta \lesssim \times 10^{-5} M_P$ is found. Thus the simple generic scenario (with a single scale set by the CMB) is not possible if the decay of the inflaton to inflatino + gravitino occurs unimpeded.

Of course, as noted in [5], if it should happen that the decay is kinematically forbidden ($|m_\phi - m_{\tilde{\phi}}| < m_{3/2}$), then the simple single scale inflationary model works very well. Here, we note that if the scale of supersymmetry breaking, μ , is significantly below that of inflation, i.e., $\mu \ll \Delta$, then even though the decay $\phi \rightarrow \tilde{\phi} + \tilde{G}$ is allowed, it will be naturally kinematically suppressed with respect to the other decay channels of the inflaton field. This will open an allowed window for Δ even in the simplest scenarios.

Independent of the details of the decay, the rate will always carry a final state momentum suppression factor. The overall decay rate can be written as $\Gamma \sim (1/m_\phi) |\mathcal{M}|^2 (p/m_\phi)$, where $|\mathcal{M}|$ is the amplitude for decay and the final state momentum suppression factor is

$$2p/m_\phi = \left(1 - \frac{2(m_{\tilde{\phi}}^2 + m_{3/2}^2)}{m_\phi^2} + \frac{(m_{\tilde{\phi}}^2 - m_{3/2}^2)^2}{m_\phi^4} \right)^{1/2} \sim \frac{\mu^2}{\Delta^2} \quad (10)$$

recalling that $m_\phi \sim m_{\tilde{\phi}} \sim \Delta^2/M_P$ and $m_{3/2} \sim \mu^2/M_P$. Thus in models in which $\mu \ll \Delta$, there will be a significant suppression in the production of gravitinos by either inflaton or inflatino decay (note that additional suppression may come from the specific form of the amplitude \mathcal{M} as well).

If we take into account the suppression factor (10) in the bound for the direct gravitino production by inflaton decay, and we combine it with the limit coming from the thermal production, we find that the scale Δ must lay within the interval

$$10^{13} \frac{\mu^2}{NM_P} \lesssim \Delta \lesssim 10^{15} \text{ GeV} \quad (11)$$

For $\mu \sim 10^{-8} M_P$, the *lower* bound in (11) is about 10^{13} GeV . For smaller values of Δ the kinematical suppression factor (10) is no longer capable of maintaining a low gravitino abundance. We notice that the scale $\Delta \sim 10^{-4} M_P$ as given in eq. (5) is within the allowed interval. Indeed, including the suppression factor (10) to eq. (8), the gravitino abundance produced by decays is now

$$Y_{3/2} \sim \frac{\Delta}{NM_P} \frac{\mu^2}{\Delta^2} \sim \frac{m_{3/2}}{N\Delta} \sim 10^{-14} \quad (12)$$

and clearly satisfies the bound (1). The simplest models of single scale inflation with purely gravitational decays and a scale given by (5), therefore, do not suffer from a gravitino problem.

For completeness, we note that when non-gravitational decays are also allowed, the kinematic suppression allows a wide range of values for Δ (and g). While the constraint from the thermal production of gravitinos is unchanged ($g\Delta < 10^9$ GeV), the constraint from decays becomes $\Delta\mu^2/gNM_P^3 < 10^{-13}$ which for $\mu = 10^{-8}M_P$ becomes $\Delta < 10^{23}g$ GeV.

Before we move on to specific examples of inflationary models, we note that there are at least two other potential sources of gravitino production. First, there is the possibility for an additional contribution to the direct production of gravitinos by the decay $\phi \rightarrow \tilde{G}\tilde{G}$. During inflaton oscillations, the rate for such decays has been estimated to be $\Gamma_{3/2} \sim \Delta^{10}/M^9$ [18]. This implies that the abundance of gravitinos will be $Y_{3/2} \sim (\Gamma_{3/2}/\Gamma)n_\phi/s \sim (\Delta/M_P)^5$. For all of the models being discussed, this is sufficiently small. Second, the remaining scalar degree of freedom in the inflaton supermultiplet may also be problematic in certain cases. This field, the sinflaton ϕ' , is normally described by the complex direction of the full inflaton potential. The complex direction is stable in most models, and thus classically it is not excited. However, if lighter than the inflatino, inflatino decay to the sinflaton + gravitino may yield another source of gravitino production. The above arguments would therefore also apply to these decays as well. In addition, it is possible that quantum fluctuations will excite the sinflaton during inflation. Even though the sinflaton is far from being massless, quantum fluctuations will lead to $\phi' \sim H$ [19]. However, in generic models since the amplitude of sinflaton oscillations is much smaller than the corresponding amplitude for inflaton oscillations, their decay into inflatinos is expected to be suppressed by a factor of $(H/M_P)^2 \sim (\Delta/M_P)^4$.

4 Lowering the scales of the inflationary sector

As we have seen, the potential of the inflaton field during inflation is constrained by the magnitude of density fluctuations measured by COBE. In the slow roll regime ($3H\dot{\phi} \simeq -V'$) eq. (4) gives

$$V_H^{1/4} \simeq 0.027 \epsilon_H^{1/4} M_P \quad (13)$$

where $\epsilon^2 \equiv M_P^2 (V'/V)^2/2 \ll 1$ is one of the two slow-roll parameters (prime denoting derivative with respect to ϕ) and the suffix H reminds us that the two quantities have to be evaluated when the scales measured by COBE left the horizon, about 60 e-foldings before the end of inflation. As we have said, in the simplest models of single scale inflation, this relation fixes the scale of the inflaton potential $\Delta \equiv V_H^{1/4}$ to be about $10^{-4}M_P$. However, models with a much smaller scale and acceptable density fluctuations can be derived. As follows from eq. (13), in models with one single field this can be done at the expense of

a small ϵ parameter, that is by taking a very flat potential during inflation. Such a flat potential may arise more naturally if more scalar fields are present, as for example in hybrid inflationary models [20]. Due to this freedom, most of the above results of the previous section have been given for an arbitrary inflationary scale.

In this section, Δ^4 denotes the value of inflaton potential at the end of inflation, when the reheating stage begins. Due to the slow motion of ϕ during inflation, this scale is typically very close to V_H . During reheating, the inflaton field oscillates about the minimum ϕ_0 of V , with $V(\phi_0) = 0$. As is typical for a massive inflaton, we assume that the quadratic term dominates the Taylor expansion of V around ϕ_0 . We denote by F the amplitude of the inflaton oscillations at the initial time $t \sim H^{-1} \sim M_P/\Delta^2$. While in the above discussions the natural assumption $F = M_P$ (originally dubbed primordial inflation [21]) was made, in the following we discuss how the results of the previous section are affected when the value of F is lower.

This analysis is particularly simplified by noticing that most of the bounds previously discussed are directly related to the inflaton mass m_ϕ rather than to the scale Δ . For a quadratic potential and generic values of Δ and F one has $m_\phi \simeq \Delta^2/F$.

Let us first consider the case in which the inflaton decays only gravitationally. For a quadratic potential, the reheating temperature is given by $T_R \simeq (\Gamma M_P)^{1/2} \simeq \sqrt{m_\phi^3/M_P}$, so that the thermal bound (3) simply gives

$$m_\phi \lesssim 10^{12} \text{ GeV} \quad (14)$$

Following the same line of arguments of the previous sections, the inflaton “abundance” at the decay time can be estimated to be

$$\frac{n_\phi}{\rho_\phi^{3/4}} \simeq \frac{T_R}{m_\phi} \simeq \sqrt{\frac{m_\phi}{M_P}} \quad (15)$$

The abundance of inflatinos produced nonthermally by the inflaton oscillations is instead (also evaluated at the inflaton decay)

$$\frac{n_{\tilde{\phi}}}{\rho_\phi^{3/4}} \simeq 10^{-2} \frac{m_\phi T_R}{M_P^2} \simeq 10^{-2} \left(\frac{m_\phi}{M_P} \right)^{5/2} \quad (16)$$

We see that the inflaton abundance is always higher than the inflatino abundance. Also the abundance of gravitinos produced nonthermally becomes smaller as m_ϕ decreases. Indeed, gravitinos will still be mainly produced at the time $t \sim m_{3/2}^{-1}$ with a typical momentum $k \sim m_{3/2}^{-1}$, independent of the value of m_ϕ . However, a lighter inflaton implies a longer life-time τ_ϕ . Since the quantity $n_{3/2}/\rho_\phi^{3/4}$ decreases in the time interval $m_{3/2} < t < \tau_\phi$, lowering m_ϕ will thus decrease the final nonthermal gravitino abundance.

Direct gravitino production still mainly occurs through the $\phi \rightarrow \tilde{\phi} \tilde{G}$ decay, if kinematically allowed. This production is reduced to a safe level as long as

$$m_\phi^{1/2} \gtrsim \frac{10^{13}}{M_P^{1/2}} \frac{\mu^2}{N M_P} \quad (17)$$

where the kinematical suppression factor $m_{3/2}/m_\phi$ has been included.

Rewriting eqs. (14) and (17) in terms of Δ and F , the allowed window for Δ becomes

$$10^{13} \frac{\mu^2}{N M_P} \sqrt{\frac{F}{M_P}} \lesssim \Delta \lesssim 10^{15} \text{GeV} \sqrt{\frac{F}{M_P}} \quad (18)$$

For $F = M_P$, this coincides with the result (11) obtained in the previous section. For smaller F , smaller values of Δ must be considered, as it is obvious from the scaling $m_\phi \simeq \Delta^2/F$.

As before, additional freedom in the choice of parameters is allowed if the inflaton has non-gravitational decays. For $\Gamma \sim g^2 m_\phi$ and for $m_{3/2} \simeq 100 \text{ GeV}$, the bounds on the reheating temperature and on the direct gravitino production become

$$m_\phi \lesssim \text{GeV}/g^2, \quad m_\phi \lesssim 10^{28} \text{GeV} g^2 \quad (19)$$

Also in this case, the bounds can be written just in terms of the inflaton mass. Notice that when the inflaton has nongravitational decays, the limit from direct gravitino production gives a lower bound (rather than an upper one) on Δ . This follows from the different scalings of the gravitational and the non gravitational rates with m_ϕ .

5 Models

Given the above arguments for the postinflationary production of gravitinos through thermal and non-thermal effects, we will now apply them to some specific examples. First we consider a simple model of inflation based on N=1 supergravity [22, 23]. We subsequently consider gravitino production in models of chaotic inflation [24]. In both examples, we will assume a superpotential of the form

$$\begin{aligned} W_T &= W(\Phi) + \mu^2 (S + \beta) \\ W(\Phi) &\equiv \Delta^2 \widetilde{W}(\Phi) \end{aligned} \quad (20)$$

where we set $M_P = 1$ for convenience. The first term specifies the dynamics of the inflaton field ϕ (i.e. the scalar component of Φ) during inflation and the first stages of reheating. The second term is known as the the Polonyi superpotential [25], and is the simplest example of

gravitationally mediated supersymmetry breaking involving only one chiral multiplet S . We also assume a minimal Kähler potential $G = K + \ln |W_T|^2$, with $K = \Phi^\dagger \Phi + S^\dagger S$. In the minimum of the theory, the Polonyi scalar takes the vacuum expectation value, $\langle s \rangle = \sqrt{3} - 1$, up to possible $\hat{\mu}^2 \equiv \mu^2/\Delta^2 \ll 1$ corrections which may arise from the interaction with the field ϕ . The parameter β is fine-tuned to $2 - \sqrt{3}$ (again up to possible $\hat{\mu}^2$ corrections) to cancel the cosmological constant.

In the following examples, we will assume that in the limit of $\mu \rightarrow 0$, supersymmetry is unbroken, that is, supersymmetry is mainly (in some cases completely) broken by the Polonyi field. Therefore, we have $F_\phi \leq \hat{\mu}^2 F_s$, where, for a minimal Kähler potential, $F_i \equiv e^{G/2} G_i = e^{K/2} (\partial W_T / \partial \varphi_i + \varphi_i^* W_T)$ and $G_i = \partial G / \partial \varphi_i$. We also have $W(\phi) \leq W(s) \hat{\mu}^2$. From the specific form of the Polonyi superpotential one finds $F_s = \sqrt{3} \mu^2$, and the gravitino mass will be

$$m_{3/2} \sim e^{K/2} \mu^2 \quad (21)$$

These relations follow from the fact that in these models, $\widetilde{W}(\phi)$ and F_ϕ/Δ^2 are at most of order $\hat{\mu}^4$ in the global minimum of the theory.

With these assumptions, the real and the imaginary components of the Polonyi field $s = (s_1 + i s_2) / \sqrt{2}$ have masses

$$m_{s_1} \sim e^{K/2} \mu^2 \sqrt{2\sqrt{3}} \quad , \quad m_{s_2} \sim e^{K/2} \mu^2 \sqrt{4 - 2\sqrt{3}} \quad (22)$$

Analogously, for the inflaton sector we decompose $\phi = (\phi_1 + i \phi_2) / \sqrt{2}$ and we denote the inflatino field by $\tilde{\phi}$. Then, a simple calculation gives (assuming $\widetilde{W}' \leq \hat{\mu}^2 \widetilde{W}'' \neq 0$ where prime denotes derivative with respect to ϕ)

$$\begin{aligned} m_{\phi_1, \phi_2} &= m_{\tilde{\phi}} \pm m_{3/2} \left(1 - \frac{\sqrt{3}}{2} \right) + \mathcal{O}(\hat{\mu}^4) \\ \frac{m_{\tilde{\phi}}}{\Delta^2} &= e^{K_0/2} \left[\widetilde{W}'' - \hat{\mu}^2 \left(\frac{\widetilde{W}'}{\hat{\mu}^2} \right)^2 + \hat{\mu}^2 \frac{K_1}{2} \widetilde{W}'' \right] + \mathcal{O}(\hat{\mu}^4) \end{aligned} \quad (23)$$

where K_0 and K_1 are the first two terms in the expansion of the function $K = K_0 + \hat{\mu}^2 K_1 + \dots$.

Note that the masses of the inflatino and of the two inflaton components are nearly degenerate, their difference being related to the small gravitino mass $m_{3/2}$. This is a consequence of having taken the supersymmetry breaking scale μ much smaller than the inflationary scale Δ . We notice that the specific choice of the Polonyi superpotential (20) gives $m_{\phi_2} \lesssim m_{\tilde{\phi}} \lesssim m_{\phi_1}$, but that the mass differences are always smaller than the gravitino mass $m_{3/2}$. Hence, the two potentially dangerous decay channels $\phi \rightarrow \tilde{\phi} + \tilde{G}$ and $\tilde{\phi} \rightarrow \phi + \tilde{G}$ are kinematically forbidden in these models.

It is important to stress that the above relations (23) are unaffected by the oscillations of the Polonyi field about its minimum. Indeed, for $m_{3/2} \simeq 100$ GeV we have

$$\frac{\Gamma_{\text{grav}}}{m_{3/2}} \sim 10^{-8} \left(\frac{\Delta}{10^{-4} M_P} \right)^6 \quad (24)$$

where $\Gamma_{\text{grav}} \sim m_\phi^3/M_P^2$ is the rate at which the decay $\phi \rightarrow \tilde{\phi} + \tilde{G}$ would occur if not kinematically suppressed or forbidden. The oscillations of the Polonyi field start at the time $t = m_{3/2}^{-1}$ and are eventually damped by the expansion of the Universe. For $t > m_{3/2}^{-1}$, their amplitude scales inversely with time. Since the initial amplitude of the oscillations and the value of the Polonyi field in the minimum are of the same order (M_P), eq. (24) shows that these oscillations can be soon neglected in the calculation of the mass spectrum.

Actually, in the following we will ignore the evolution of the Polonyi field. This evolution, however, leads to a well known problem in cosmology which is compounded in more modern theories collectively called the moduli problem. [26]

5.1 New inflation

A workable example of inflation is provided by [23]

$$W(\Phi) = \frac{\Delta^2}{2 M_P} (\Phi - \phi_0)^2 \quad (25)$$

which is clearly of the form advocated in section 2.

During inflation and the first stages of reheating, the Polonyi scalar is frozen at $s = 0$. Moreover, since $\mu \ll \Delta$, the Polonyi sector is completely negligible at this stage. Thus, for the moment, we can set $\mu = 0$ and decompose the inflaton field into its real plus imaginary components, $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$. Inflation occurs while $\phi_1 \simeq 0$. Near the origin, the real direction is relatively flat, the imaginary component has large positive mass, and it is rapidly driven to zero. At the classical level, we can set $\phi_2 \equiv 0$, since V'' is always positive in the imaginary direction. Doing so, one finds

$$V = \frac{\Delta^4}{4} e^{\phi_1^2/(2M_P^2)} \left(\frac{\phi_1}{\sqrt{2} M_P} - 1 \right)^2 \left[1 + \sqrt{2} \frac{\phi_1}{M_P} + \frac{\phi_1^2}{M_P^2} - \frac{\phi_1^3}{\sqrt{2} M_P^3} + \frac{\phi_1^4}{4 M_P^4} \right] \geq 0 \quad (26)$$

where $\phi_0 = M_P$ has been set to have a vanishing cosmological constant in the minimum at $\phi_1 = \sqrt{2} M_P$. Notice that in the vicinity of the origin, both the linear and quadratic terms in ϕ_1 cancel when the exponential prefactor is expanded. Inflation in this model occurs for $\phi_1 \lesssim M_P/(6\sqrt{2})$. The inflationary scale Δ which matches the microwave background anisotropies is about $\Delta/M_P \simeq 2.6 \times 10^{-4}$, and the spectral index is found to be $n_s \sim 0.93$.

Let us now consider the late behavior of the fields and reintroduce the dimensionless quantity $\hat{\mu} \equiv \mu/\Delta \ll 1$. The presence of a nonvanishing Polonyi field slightly modifies the potential for ϕ and the position of the minimum. Also, the nonvanishing vev of ϕ forces a small modification to the Polonyi potential. These effects, which are easily computed in an expansion series in $\hat{\mu}^2$, must be taken into account in the computation of the spectrum in the minimum of the theory. There is some freedom in the choice of the parameters, since different combinations of ϕ_0 and β give a zero cosmological constant. Up to order $\hat{\mu}^6$ corrections, one finds

$$\begin{aligned}\frac{\phi_0}{M_P} &= 1 + f \hat{\mu}^2 + g \hat{\mu}^4 \\ \frac{\beta}{M_P} &= (2 - \sqrt{3}) - \hat{\mu}^2/2 + \left(\frac{3}{4} - \frac{\sqrt{3}}{2} - f\right) \hat{\mu}^4\end{aligned}\tag{27}$$

with f and g arbitrary real numbers.

Decomposing $s = (s_1 + i s_2)/\sqrt{2}$, one finds $s_2 \equiv 0$, while the real components have their minima at ¹

$$\begin{aligned}\frac{\langle \phi_1 \rangle}{\sqrt{2} M_P} &= 1 + (f - 1) \hat{\mu}^2 + \left(g - f + \frac{1}{2} - \sqrt{3}\right) \hat{\mu}^4 \neq \phi_0 \\ \frac{\langle s_1 \rangle}{\sqrt{2} M_P} &= \sqrt{3} - 1 + \frac{\hat{\mu}^2}{2} + \left(f + \frac{\sqrt{3}}{2} - \frac{9}{8}\right) \hat{\mu}^4\end{aligned}\tag{28}$$

again up to order $\hat{\mu}^6$ corrections.

In the minimum, the F -terms are given by

$$\frac{F_s}{\Delta^2} \simeq \sqrt{3} \hat{\mu}^2 + \frac{\sqrt{3}}{2} \hat{\mu}^4, \quad \frac{F_\phi}{\Delta^2} \simeq -\sqrt{3} \hat{\mu}^4\tag{29}$$

Hence, the inflaton sector provides a negligible (although non-vanishing) contribution to supersymmetry breaking in the minimum of the theory. At late times the goldstino can be identified with the Polonyi fermion, up to small $\hat{\mu}^2$ corrections. The masses of the gravitino and of the two components of the Polonyi field are given by the general expressions (21) and (22), so that we have

$$\hat{\mu}^2 \simeq 2.8 \cdot 10^{-10} \left(\frac{m_{3/2}}{100 \text{ GeV}} \right)\tag{30}$$

¹For these minima one has

$$\frac{\partial V}{\partial \phi} = \mathcal{O}(\hat{\mu}^8), \quad \frac{\partial V}{\partial s} = \mathcal{O}(\hat{\mu}^{10}), \quad V = \mathcal{O}(\hat{\mu}^{10})$$

The general relations (23) for the inflationary sector are also respected. More explicitly, one finds

$$\begin{aligned} m_{\tilde{\phi}} &= \frac{\Delta^2}{M_P} e^{\frac{5}{2}-\sqrt{3}} \left[1 + \left(f + \frac{\sqrt{3}}{2} - \frac{5}{2} \right) \hat{\mu}^2 \right] \\ m_{\phi_1} &= \frac{\Delta^2}{M_P} e^{\frac{5}{2}-\sqrt{3}} \left[1 + \left(f - \frac{3}{2} \right) \hat{\mu}^2 \right] \\ m_{\phi_2} &= \frac{\Delta^2}{M_P} e^{\frac{5}{2}-\sqrt{3}} \left[1 + \left(f - \frac{7}{2} + \sqrt{3} \right) \hat{\mu}^2 \right] \end{aligned} \quad (31)$$

confirming that the two channels $\phi_1 \rightarrow \tilde{\phi} + \tilde{G}$ and $\tilde{\phi} \rightarrow \phi_2 + \tilde{G}$ are indeed kinematically forbidden.

With regard to the nonthermal production of inflatinos and gravitinos, we have verified by numerical calculations that the model (25) gives results in very good agreement with the ones obtained in [8] for the chaotic inflationary superpotential $W(\Phi) = m_\phi^2 \Phi^2/2$ (see below). Inflatinos are produced during the first oscillations of the inflaton field ($t \sim m_\phi^{-1} = M_P/\Delta^2$). Their final abundance $Y_{\tilde{\phi}}$ is given in equation (7). As we have remarked, the final value of $Y_{\tilde{\phi}}$ is typically very small, due to the dilution of $n_{\tilde{\phi}}/\rho_\phi$ during the inflaton oscillations in the massive inflaton case. The nonthermal production of gravitinos is instead mainly due to the Polonyi oscillations, and indeed it occurs on a timescale $t \sim m_s^{-1} \sim m_{3/2}^{-1}$. As we have already mentioned, their final abundance is much smaller than that of inflatinos, since they are typically produced with a physical momentum $k \sim m_{3/2}$.

5.2 Chaotic inflation

For completeness, we consider two types of chaotic models with superpotentials

$$W(\Phi) = \frac{1}{2} m_\phi \Phi^2 \quad \text{and} \quad W(\Phi) = \frac{\lambda}{3} \Phi^3 \quad (32)$$

which for $\phi \ll M_P$ lead to the usual quadratic and quartic potentials typical of chaotic inflation [24]. As it is well known, the above superpotentials do not lead to inflation in supergravity, since corrections from the Kähler potential spoil the flatness of $V(\phi)$ for $\phi \gtrsim M_P$ (at least in minimal supersgravity). One can still assume that some corrections, relevant at high ϕ , will generate a sufficiently flat potential so to render inflation possible, and that the superpotentials (32) will be accurate enough during the reheating stage (see [27] for example, for chaotic models fully consistent with supergravity). After inflation the production of gravitinos can be very well estimated with the superpotential (32), at $\phi \ll M_P$ [8]. However, the two parameters m_ϕ and λ cannot be directly linked to the size of the density fluctuations

as in eq. (4), and the “standard” values $m_\phi \sim 10^{13}$ GeV and $\lambda \sim 10^{-13}$ should be taken only as indicative ones. In both cases, $\phi = 0$ is the minimum of the model, and the inflaton sector does not contribute to supersymmetry breaking ($F_\phi = 0$) at late times.

Let us first consider the superpotential $W = m_\phi \Phi^2/2$. Evaluation of eqs. (23) for the masses of the inflaton components simply gives

$$m_{\tilde{\phi}} = e^{2-\sqrt{3}} m_\phi \quad , \quad m_{\phi_1, \phi_2} = e^{2-\sqrt{3}} \left[m_\phi \pm \frac{\mu^2}{M_P} \left(1 - \frac{\sqrt{3}}{2} \right) \right] \quad (33)$$

The nonthermal production of inflatinos and gravitinos in this model has been studied in [8]. The results have been briefly summarized in section 2 and at the end of the previous subsection.

Let us now consider the superpotential $W = \lambda \Phi^3/3$. In this model as well, the nonperturbative production of inflatinos and gravitinos is analogous to the ones already discussed. Inflatinos are mainly produced during the first few oscillations of the inflaton field, with a typical momentum [7] $k \sim m_\phi \sim \sqrt{\lambda} \phi_0$, $\phi_0 \sim M_P$ being the initial amplitude of the inflaton oscillations. There is however an important difference, due to the fact that, during the oscillations, in the $V(\phi) \sim \phi^4$ model the inflaton field has the equation of state of radiation, and its energy density scales as R^{-4} . Thus the energy density of ϕ oscillations is given by

$$\rho_\phi \sim \lambda \phi_0^4 (R_\phi/R)^4 \quad (34)$$

In this phase both $n_{\tilde{\phi}}$ and $s \equiv \rho_\phi^{3/4}$ scale as R^{-3} . The dilution factor then drops out from their ratio (cf. the last factor in eq. (7)) and one is simply left with

$$\frac{n_{\tilde{\phi}}}{s} \sim 10^{-2} \frac{(\sqrt{\lambda} \phi_0)^3}{(\lambda \phi_0^4)^{3/4}} \sim 10^{-12} \left(\frac{\lambda}{10^{-13}} \right)^{3/4} \quad (35)$$

As for the previous cases, nonthermal gravitino production is expected to occur mainly during the oscillations of the Polonyi field, with a negligible final abundance.

In the minimum of $V(\phi)$ the masses of the two inflaton components, as well as that of the inflatino, vanish, although the inflaton is still massive during its oscillations. Since the amplitude ϕ changes very slowly with respect to the oscillations timescale, we can get the inflaton mass from the average of $\sqrt{d^2V/d^2\phi}$ during one oscillation. Up to a numerical factor of order one, we can set

$$m_\phi(t) \sim \sqrt{\lambda} \phi(t) \quad (36)$$

It is interesting to note that in this model the inflaton cannot decay gravitationally, since with the above mass one has

$$\Gamma_{\text{grav}} \simeq \frac{m_\phi^3}{M_P^2} \simeq \frac{\lambda^{3/2} \phi_0^3}{M_P^2} \left(\frac{R_\phi}{R} \right)^3 \quad (37)$$

This should be compared to the Hubble rate

$$H \sim \frac{\lambda^{1/2} \phi_0^2}{M_P} \left(\frac{R_\phi}{R} \right)^2 \quad (38)$$

Clearly the gravitational decay rate is always smaller than H . For a non-gravitational decay with a rate $\Gamma = g^2 m_\phi \sim g^2 \lambda^{1/2} \phi_0 (R_\phi/R)$, which should be compared to eq. (38). Thus the scale factor at decay is $(R_\phi/R_{d\phi}) \sim M_P g^2 / \phi_0$ and the inflaton lifetime is given by

$$t \sim \frac{M_P^{-1}}{\lambda^{1/2} g^4} \sim m_{3/2}^{-1} \left(\frac{4 \cdot 10^{-3}}{g} \right)^4 \quad (39)$$

compared here with the timescale of the Polonyi oscillations. Thus although the inflatino abundance in eq. (35) is somewhat high, its decay (which is necessarily non-gravitational) does not lead to gravitino production.

6 Discussion

The simple models discussed in this letter were chosen to highlight the potential problems induced by the inflatino. While more general remarks have been made in earlier sections, explicit computations have been performed for a single inflaton supermultiplet along with the supersymmetry breaking sector described by a Polonyi superfield. As remarked earlier, we have ignored the cosmological problems associated directly with the Polonyi scalar's evolution. We have also neglected the potentially important evolution of other scalar fields or other sources of entropy production.

Since the MSSM contains many flat directions, the cosmological evolution along these directions can have a direct impact on the issues discussed here. For example, it is well known that some flat directions may be responsible for generating the baryon asymmetry of the Universe [28]. In Affleck-Dine baryogenesis scenarios, if the initial field values of the flat direction is larger than a critical value, $\eta_0 \gtrsim m_{3/2}^{5/12} M_P^{4/3} / m_\phi^{3/4}$, where η is the A-D condensate, the dominant source of entropy comes from η decays rather than inflaton decays, although in this case, the baryon asymmetry is generally too large. For smaller initial values of η_0 , the A-D condensate decays during inflaton oscillations. Depending on the value, η_0 , late entropy production by moduli decay may be a welcome feature to bring the baryon asymmetry down [29].

We have shown that the decay of inflatons to/from inflatinos + gravitino is potentially a serious problem for a generic inflationary model based on supergravity. This problem is particularly enhanced when the inflaton sector is coupled only gravitationally to matter. In

this case the decay channels with a final state gravitino can have rates comparable to the decays into matter. This existence of these gravitino producing decays depends on the mass spectrum of the model. In the particular case of Polonyi-like supersymmetry breaking, we have shown that such decays are in fact kinematically forbidden, since the mass difference between the inflaton and the inflatino is smaller than the gravitino mass.

This fortunate property is not expected to hold for a general supersymmetry breaking potential. More generally, the direct gravitino production must be forbidden by imposing a strong hierarchy between the scale of inflation Δ and the size of supersymmetry breaking. In gravitational mediated supersymmetry breaking schemes ($m_{3/2} \sim 100$ GeV) this can be translated into an upper bound on Δ . This limit weakens as the initial size of the inflaton oscillations is decreased. For initial oscillations of the order the Planck scale (primordial inflation) the bound is quite severe, and a safe value for the inflationary scale is $\Delta \gtrsim 10^{13}$ GeV. Models of single scale inflation with a scale dictated by the size of the density fluctuations are typically compatible with this bound.

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